$$= \lim_{x \to 0} x = 0$$

Both functions go to zero as you take the limit. So then the function is:

$$xsin(\frac{1}{x})$$

Goes to zero when you take the limit because of the Squeeze Theorem.

This is really all for the Squeeze Theorem. You may get an exam problem on it and some homework problems, but you will not need it to take derivatives, so it is an offshoot of the main topics in calculus one. It is really an interesting theorem, and it makes sense in practice, but it can be tricky to use. So, take your time when you must use the Squeeze Theorem. I suggest you do a couple of problems. I would do more then what I have shown you and your textbook will have some more sections on this topic if your not feeling comfortable.

Definition of a Derivative

Finally, after all the information about limits we get to taking a derivative. Don't get too excited by this, the definition of a derivative is all algebra and a lot of it. Yes, taking a derivative, the normal way of a polynomial function is really easy, and your math teacher will make you take the definition of a derivative the long way for at least a third of the semester. So, odds are if you struggle with taking the definition of a derivative you will have to learn it and it is almost 100% the algebra with which you are struggling. Go back to the basics and sharpen your algebra skills. There will be more problems in the back of the book to sharpen your algebra skills, specifically for taking the definition of a derivative.

You know what the idea of a derivative is and what limits are from earlier in this section of the book, but what exactly is the definition of a derivative? Well, as stated before, a derivative is the instantaneous rate of change at a point from a given function. Now, how is this done, especially by using the following definition for a derivative? Well, we have a formula, and we use the concept of limits that I will expand upon after I show you the formula. See the definition of a derivative formula below.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The notation for $f'(x)$ is the notation stating that the given function is a derivative of the original function. You have $f(x)$ then you take the derivative of the function, and it then becomes $f'(x)$. You may also see $\frac{dy}{dx}$ this is the same as, $f'(x)$ however, you will see it more with implicit differentiation, which will be covered later. The next part of the definition of the derivative is inside the limit statement. Where $f(x+h)$ is the function given plus *h*.

For example, if:

$$f(x) = 4x^2 + 2x + 3$$

Then:

$$f(x + h) = 4(x + h)^2 + 2(x + h) + 3$$

Hopefully, the above statement clears up what $f(x + h)$ is. The h is used so you can plug in the limit statement later, after some algebra is applied to the statement.

We know that you add the h to the statement. But why do we add it? Well, the h represents a small change in x. This allows for a very small shift in the value of x instead of the larger change shown with algebra in the average rate of change function. See the formula below for the average rate of change.

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_2}$$

Where $\frac{\Delta y}{\Delta x}$ is a bigger change (average rate of change) compared to $\frac{dy}{dx}$ is a smaller (instantaneous rate of change).

A better way of thinking of both the algebra and calculus versions is that the algebra version is that you take a given function and two separate points and find the difference between these two points. Thus, it is why it's the average rate of change because the function could vary a lot more with respect to the two points. Think of you having a point on one side of the graph and then a point on another side, and the function can go crazy between them, but the function will average out.

The value of *h* in the definition of a derivative represents this change of the two points, except it's a very small change. Now it is not an average rate of change; it is an instantaneous rate of change. I want you to picture a little movement along a given function where you have a value of x, then you move just a little so that your value of x is now $(x + h)$ but *h is…* basically zero. The zero is where the limit comes in to evaluate the function as *h* approaches zero. Now you're probably still wondering why you're dividing by *h*. Well, these denominator values have a change in *x,* too. Since *h* is the small change in *x*. Whereas the numerator is the small change in the function, and the limit allows you to evaluate this small change in the function. I provide a little summary of the definition of a derivative. In case the explanation is not completely clear to you.

Summary of the Definition of a Derivative:

- *h* is the small change of x at a point.
- The denominator in the definition of a derivative is the small change in *x*.
- Then the numerator of the function in the definition of a derivative is the small change of the given function that is being derived. (This can be thought of as the change in the "y" value in simpler terms, but is really the change in the function that is being derived)
- The limit is there to evaluate the slight change since h is zero, but is not actually fully zero.

Hopefully, we now understand the definition of a derivative formula, and we can start to do example problems to take the derivative of a given function with the definition. We will start with an easy example for our first problem. Remember to carry the limit statement all the way through until you actually take the limit. You will lose points on an exam or quiz if you do not keep the limit statement in the expression until you take the limit.

What you need to be able to do for this Problem:

- *Distributing Terms: Foiling and Distributing*
- *Foiling Terms: Foiling and Distributing*
- *Combine Like Terms: Combining Like Terms*
- *Factor Out Terms: Foiling and Distributing*
- *Evaluation of the Definition of a Derivative: Currently in this section*

Take the Definition of a Derivative of the following statement.

$$f(x) = x^2$$

Now, start with taking this expression and plugging the statement into the definition of a derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

You are literally plugging in the function as stated in the definition of a derivative. If this is confusing to you, it will take practice to do it correctly. This is an easier problem; however, they do get much harder. It can be some pattern recognition that you need to be able to deal with the more complex problems.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - (x^2)}{h}$$

The next step is to expand and combine like terms. This is where you need to be good at algebra.

$$= \lim_{h\to 0} \frac{(x+h)(x+h) - (x^2)}{h}$$

So, it was expanded out; now you will have to foil the terms you just expanded to simplify the statement.

$$= \lim_{h\to 0} \frac{(x^2 + xh + xh + h^2) - (x^2)}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{x^2} + xh + xh + h^2) - (\cancel{x^2})}{h}$$

$$= \lim_{h\to 0} \frac{(2xh + h^2)}{h}$$

Now we are close to taking the limit statement, but the next step is to factor out the h so that there will be nothing in the denominator since h divided by h is one.

$$= \lim_{h\to 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h\to 0}(2x+h)$$

Now we can finally plug in zero for h and take the limit to get the derivative.

$$f'(x) = 2x$$

You have successfully solved your first derivative. You may have had some problems. If you did, please refer to the summary at the beginning of the problem to pinpoint what you need to work on and go to those problems. You will be able to find more problems in the back of the book to sharpen up your skills. Additionally, you may be asking yourself if this worked out too well, and how Issac Newton made the connection between the two types of mathematics? I do not have the answer to this question. I believe even if you gave me all the pieces, I don't think I would have put this together as Issac Newton did. Nevertheless, let's keep going and take more derivatives.

What you need to be able to do for this Problem:

- *Foil the Expanded Expression: Foiling and Distributing*
- *Combine Like Terms: Combining Like Terms*
- *Factor Out Terms: Foiling and Distributing*
- *Cross Out Like Terms If they are in both the Numerator and the Denominator: Fractions*
- *Evaluation of the Definition of a Derivative: Currently in this section.*

Take the Definition of a Derivative of the following statement.

$$f(x) = 3x^2$$

Now, start by taking this expression and plugging the statement into the definition of a derivative.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

You are literally plugging in the function as stated in the definition of a derivative. Then you will need to apply the algebra to solve the rest of the problem. Additionally, the last step will be evaluating the limit, and this is always the case when you are using the definition of a derivative.

$$f'(x) = \lim_{h\to 0}\frac{3(x+h)^2-3(x^2)}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{3x^2}+6xh+3h^2-\cancel{3x^2}}{h}$$

$$= \lim_{h\to 0}\frac{6xh+3h^2}{h}$$

$$= \lim_{h\to 0} 6x+3h$$

$$= 6\mathrm{x}+3(0)$$

$$= 6\mathrm{x}$$

What you need to be able to do for this Problem:

- *Foil the Expanded Expression: Foiling and Distributing*
- *Combine Like Terms: Combining Like Terms*
- *Factor Out Terms: Foiling and Distributing*
- *Cross Out Like Terms If they are in both the Numerator and the Denominator: Fractions*
- *Evaluation of the Definition of a Derivative: Currently in this section.*

Take the Definition of a Derivative of the following statement.

$$f(x) = 3x^2 + 4x$$

Now, start by taking this expression and plugging the statement into the definition of a derivative. I am going to be less involved. I would like you to try and solve this problem on *your own. It will be more difficult but only refer to this for the answer or if you get stuck. Also, maybe do this problem a couple of times before you go to the next one if you end up stuck.*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 + 4(x+h) - (3x^2 + 4x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 4x + 4h - 3x^2 - 4x}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 4x + 4h - 3x^2 - 4x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{4x} + 4h - \cancel{3x^2} - \cancel{4x}}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2 + 4h}{h}$$

$$= \lim_{h \to 0} \frac{3h^2 + 6xh + 4h}{h}$$

$$= \lim_{h \to 0} 3h + 6x + 4$$

$$= 3(0) + 6x + 4$$

$$= 6x + 4$$

What you need to be able to do for this Problem:

- *Foil the Expanded Expression: Foiling and Distributing*
- *Combine Like Terms: Combining Like Terms*
- *Factor Out Terms: Foiling and Distributing*
- *Cross Out Like Terms If they are in both the Numerator and the Denominator: Fractions*
- *Multiply by the Conjugate: Multiplying by the Conjugate*
- *Evaluation of the Definition of a Derivative: Currently in this section.*

Take the Definition of a Derivative of the following statement.

$$f(x) = \sqrt{x}$$

Now, start by taking this expression and plugging the statement into the definition of a derivative.